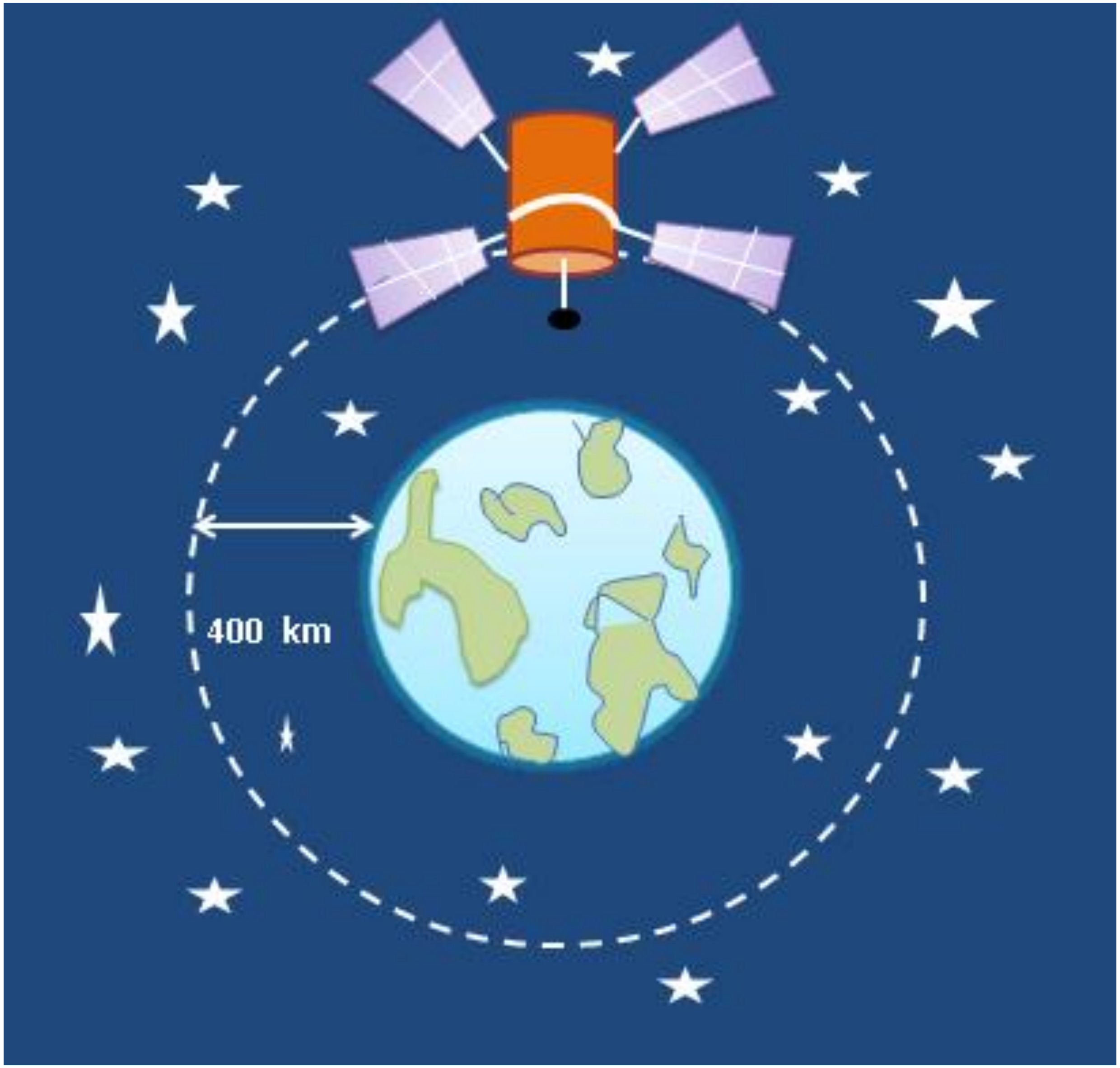
PROBLEM BASED ON

GRAVITATION

Question

A satellite of mass 200 kg orbits the earth at a height of 400 km above the surface. How much energy must be expended to rocket the satellite out of the earth's gravitational influence?


400 km

SOLUTION:

Given:

Mass of the satellite,

m = 200 kg

Height of the satellite,

x = 400 km

$= \mathbf{0.4 \times 10^6} \boldsymbol{m}$

As we know,

Mass of the earth, M = $\mathbf{6.0 \times 10^{24}}\ \boldsymbol{kg}$**;**

Radius of the earth, R = $\mathbf{6.4 \times 10^{6}}\boldsymbol{m}$

Gravitational Constant,

$$G = 6.67 \times 10^{-11} Nm^{2}kg^{-2}$$

When a satellite is orbiting around the earth, it possess kinetic energy, K.E.= $\frac{1}{2}mv^2$

v **is the orbital velocity of a satellite which keeps the satellite into its orbit around the earth i.e.**

$$v = \sqrt{\frac{GM}{(R+x)}}$$

Then, kinetic energy becomes, K.E. = $\frac{GMm}{2(R+x)}$

Potential energy of the satellite due to its position in the gravitational field of earth at a height x above the surface of earth is given by

$$U = -\frac{GMm}{(R+x)}$$

Here negative sign shows the attractive force due to earth on the satellite.

Total energy of the satellite at height x,

$$E = K.E. + P.E.$$

$$E = \frac{GMm}{2(R+x)} + \left(-\frac{GMm}{(R+x)}\right)$$

$$E = -\frac{GMm}{2(R+x)}$$

Here, negative sign shows that satellite is bound to earth. This is called bound energy of the satellite.

Now, substituting all the values in total energy of the satellite,

$$E = -\frac{GMm}{2(R+x)}$$

$$= -\frac{6.67\times10^{-11}\times6.0\times10^{24}\times200}{2\times\left(6.4\times10^{6}+0.4\times10^{6}\right)}$$

$$= -\frac{6.67\times6.0\times2\times10^{15}}{2\times6.8\times10^{6}}$$

$$= -5.89\times10^{9}J$$

To free the satellite from the earth's gravitational field, negative of the total energy of the orbiting satellite has to be supplied.

$$\textbf{Binding energy} = -\left(-5.89 \times 10^{9}\right)$$

$$= 5.89 \times 10^{9} J$$

Thus, $5.89 \times 10^{9} J$ energy must be expended to rocket the satellite out of the earth's gravitational influence.